

**Bayesian Decision-Making
Fundamentals:
Goal-Fabric Analysis**

Thesis

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ABSTRACT

Modern decision makers are confronted with the task of making qualitative, multi-criterion decisions under uncertainty with only highly quantitative, profit oriented models available as aids in making them. Presently, there are two sources from which the decision maker can obtain information on decision making techniques. The first is highly mathematical literature on Bayesian and statistical decision theory. The second is business and technical applications of decision theory. The first source is too technical for most decision makers; the second does not present the fundamentals. This thesis project presents Bayesian decision making fundamentals in a logical, non-technical text providing the decision maker with the basics necessary to make decisions under uncertainty using both qualitative and quantitative criteria.

The project is organized into: six concept chapters, a summary, a reference guide, and four appendices. The concept chapters utilize one main example problem which is solved using each new concept. Each solution utilizes (1) the problem solution framework, i.e., problem definition, selecting objectives, synthesis, analysis, and final alternative selection, and (2) decision diagrams. Each part of the problem solution framework is defined and discussed. Linear and non-linear preference (utility) is presented. Probability is introduced as well as the concepts of uncertainty, state of nature, and expected value. Two multi-criterion analyses are outlined. One is quantitative, weighted sums of numerical criteria estimates; the other is qualitative, a goal

fabric analysis. The project also deals with the economics of additional information utilizing conditional probabilities, Bayes' rule, and a pre-posterior analysis. The appendices are: (1) a glossary of terms used in the text, (2) a table of symbols, (3) a full discussion of decision diagrams, and (4) a detailed explanation of how to assess preferences and generate preference curves.

TABLE OF CONTENTS

	Page
LIST OF FIGURES	vii
LIST OF TABLES	viii

PART I

PROJECT REPORT	1
History	2
Thesis	3
Description of Thesis Project	3
Frameworks	4
Multi-criterion analyses and additional sampling	6
Appendices	7
Use	8
Results	8
Limitations	8
Background Requirement	9
Additional Work	9
Conclusion	10

PART II

BAYESIAN DECISION MAKING FUNDAMENTALS

Chapter		
1.	INTRODUCTION	11
2.	PROBLEM SOLUTION FRAMEWORK	14
	Problem Solution	14
	Problem Definition	15

Selection of Objectives	17
Generation of Alternatives (Synthesis)	17
The Decision Diagram	18
Analysis	22
Selection of Best Alternative	23
3. PREFERENCE	24
Definition	24
Preference Functions	26
Example Problem Solution	26
Analysis	26
Selection of best alternative	27
Non-linear Preference	28
Example Problem Solution	30
Analysis	30
Selection of best alternative	32
4. UNCERTAINTY	33
Uncertain Costs and Benefits	33
Probability	33
Probability rules	34
State of Nature	34
Probability estimates of states of nature	34
Diagramming uncertain states of nature	35
Expected Value	36
Example Problem Solution	37
Decision Diagram	37
Analysis	37
Selection of Best Alternative	40

5. MULTI-CRITERION ANALYSES	41
Criteria Definitions	41
Selecting Criteria	41
Quantification	43
Relevancy	43
Multi-Criterion Analyses	43
Weighting and Summing	44
Goal Fabric	47
Summary	51
6. ADDITIONAL SAMPLING	52
Example Problem	52
Preliminary Definitions	52
Conditional probabilities	53
Prior and posterior probabilities	53
Bayes' rule	53
Outcome likelihood probabilities	54
Preposterior Analysis	55
General decision diagram	55
Preliminary information	57
Steps in a preposterior analysis	62
Analysis calculations	63
Perfect Information	65
Summary	68
7. CONCLUSION	69
Summary	69
Problem solution (Chapter 2)	69
Decision diagrams (Chapters 2 and Appendix C)	70

	Page
Criteria (Chapters 2 and 5)	70
Preference (Chapters 3 and Appendix D)	70
Multi-criterion analyses (Chapter 5)	71
Additional sampling (Chapter 6)	72
Guide to References	73
Problem solution	73
Statistics and probability	73
Fundamentals of decision theory	73
Analysis models	73
Mathematical theory	74
REFERENCES	75
APPENDICES	
A. Glossary	78
B. Table of Symbols	80
C. Decision Diagrams	81
D. Preference Assessment and Curve Generation	92

LIST OF FIGURES

Figure	Page
1. Example Area Map	16
2. Decision Diagram for Example Problem	21
3. Linear Preference Function	25
4. Risk Averse Preference Curve	29
5. Risk Seeking Preference Curve	29
6. Non-Linear Preference Function	31
7. Example of a Fan	35
8. Example Problem Decision Diagram using Uncertain Net Gains	39
9. Goal Fabric for Example Problem	48
10. General Decision Diagram of a Preposterior Analysis	56
11. Rainy Season Decision Problem Diagram	60
12. Diminishing Returns of Sampling	66
13. General Plot of Expected Return versus Sampling Length	67
14. Diagram of the Anniversary Problem	83
15. Example of an Act Fan	85
16. Examples of Event Fans	85
17. Example Problem Decision Diagram	89
18. Preference Curve	93
19. Example Problem Decision Diagram	96
20. Steps in Preference Assessment and Curve Generation .	98
21. Example Preference Curve	102
22. Example Problem Decision Diagram using Preference . .	104

LIST OF TABLES

Table	Page
1. Specifications for Alternatives	19
2. Benefits and Costs per Annum	20
3. Net Gains for Each Alternative	23
4. Analysis Summary using Linear Preference	27
5. Analysis Summary using Non-Linear Preference	30
6. Cost and Probability Estimates for Example Problem	38
7. Analysis of Example Problem using Expected Non-Linear Preference	40
8. Objectives, Criteria, and Units for the Example Problem	42
9. Criteria Estimates for Example Problem (Includes Weighting and Summing Analysis Results)	45
10. Criteria Estimates for Transportation Alternatives .	50
11. Preferences for Transportation Alternatives	51
12. Outcome Likelihood Probabilities	54
13. Experiment Costs and Outcome Likelihood Probabilities	58
14. Alternative Evaluations and Prior Probabilities . .	59
15. Posterior Probability Calculations	61
16. Calculations of Outcome Probabilities	62
17. Calculations and Results of Preposterior Analysis of Rainy Season Problem	64
18. Outcome Likelihoods for 90 and 180 days of Sampling.	67
19. Evaluation Information	95
20. Expected Value Analysis	97
21. Example Preference Assessments	102

Table 8

Objectives, Criteria, and Units
for the Example Problem

Objective	Criteria	Units
(1) Preserve forest and natural environment	slash	tons
	particulates in air and water	tons
	wildlife destroyed	percent of population
	water wasted	gallons per year
(2) Prevent erosion	earth lost	tons per year
(3) Maintain service between Logville and Lakeview	time open during construction	percent of time
(4) Involve area citi- zens in decision making process	number involved	percent
	weight in decisions	percent
(5) Maximize net gain	present worth over 20 years	dollars per day

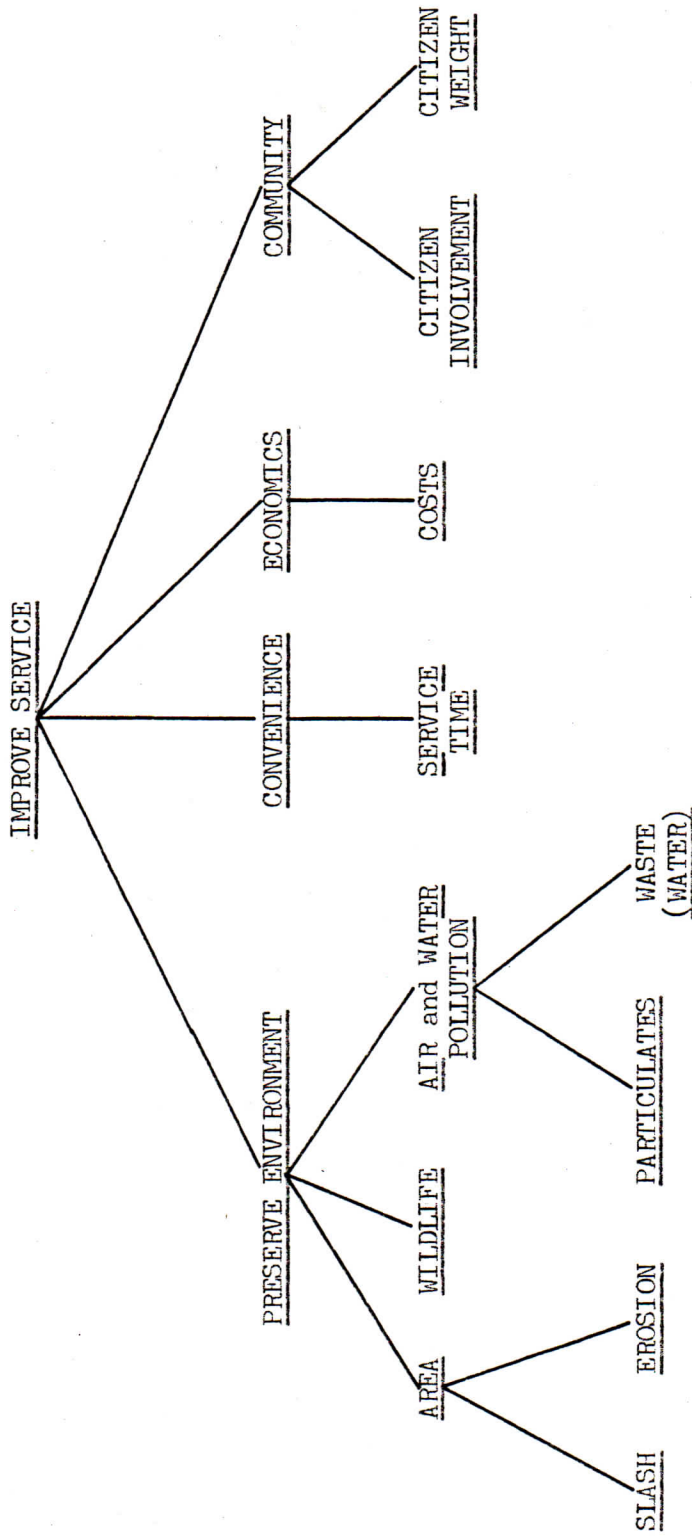


Figure 9

Goal Fabric for Example Problem

Table 11

Preferences for Transportation
Alternatives

Alternative	Criteria			Total
	Beauty	Convenience	Average Speed	
A	2.5	1.0	3.0	21
B	4.0	0.5	3.2	23
weights	2	4	4	

Summary

The above analyses provide two possible ways to make decisions using many objectives and criteria. Similar analyses are available or in the development stage and can be found in decision making literature. However, the decision maker need not rely on specific procedures to perform his analysis. By using the principles in this chapter, he should be able to design an analysis to solve his problem. The success of his analyses will improve as the decision maker practices and experiments with the fundamentals.